The Islamia University Of Bahawalpur,
Department of Computer Science & IT
Bahawalnagar Campus
Course: Numerical Analysis **Program:** BSCS-V (Spring 2020)

Topic: Experimental data method

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Polynomial Approximation - Experimental data:

"You may well have experience in carrying out an experiment and then trying to get a straight line to pass as near possible to the data plotted on graph paper.

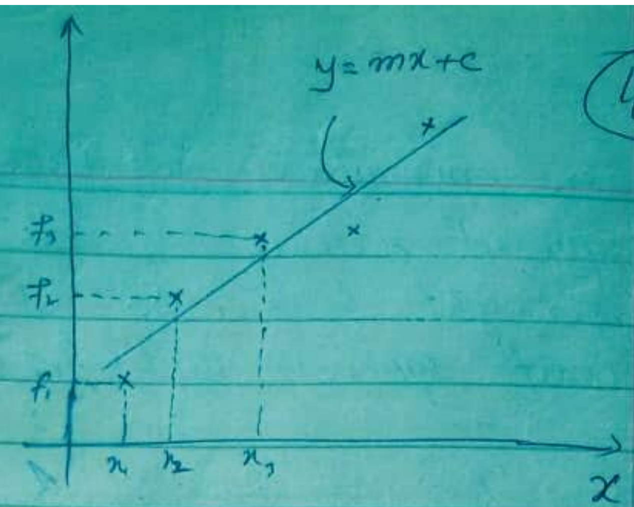
This process of adjusting a clear ruler over the page until it looks "about right" is fine for a rough approximation, but it is not especially scientific.

Any software you use which provides a "best fit" straight line must obviously employ a (less haphazard) approach.

Here we show one way in which best fit straight lines may be found.

⇒ Best Fit straight lines:

Let us consider to get a straight line "$y = mx + c$" to be as near as possible to experimental data in the form (x_1, f_1), (x_2, f_2), (x_3, f_3), ----



Our goal to minimise the overall distance between the crosses (the data points) and the straight line.
For this we adopt here involves minimising the quantity.

i.e

$$R = \underbrace{(mx_1 + c - f_1)^2}_{\text{vertical distance between line and the point } (x_1, f_1)} + \underbrace{(mx_2 + c - f_2)^2}_{\text{Second data point Distance}} + \underbrace{(mx_3 + c - f_3)^2}_{\text{For 3rd point Distance}} + \cdots$$

$$R = \sum (mx_n + c - f_n)^2 \quad \text{—— (i)}$$

Each term in the sum measures the vertical distance b/w data point and the straight line (i.e Required Straight line)

⇒ Squaring the distances ensures that distance above and below the line do not cancel out each other.

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For minimising the distances we take squared distance b/w data point and required straight line. So it called the "least squares best fit straight line"

As 'R' is function of two variables 'm' slope of line and 'c', the y-intercept of line
From the concept of Maxima, Minima of functions.
The minimisation 'R' is achieved when

$$\frac{\partial R}{\partial c} = 0 \quad \text{(ii)} \quad \text{and} \quad \frac{\partial R}{\partial m} = 0 \quad \text{(iii)}$$

'R' becomes maximum if we make it bigger by moving the line further away from data points

Differentiate 'R' w.r.t 'm' and 'c'

$$\frac{\partial R}{\partial c} = 2(mx_1 + c - f_1) + 2(mx_2 + c - f_2) + 2(mx_3 + c - f_3) + \cdots$$

$$\frac{\partial R}{\partial c} = 2\sum(mx_n + c - f_n)$$

And

$$\frac{\partial R}{\partial m} = 2(mx_1 + c - f_1)x_1 + 2(mx_2 + c - f_2)x_2 + 2(mx_3 + c - f_3)x_3 \cdots$$

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$$\frac{\partial R}{\partial m} = 2\sum(mx_n + c - f_n)x_n$$

By eq(ii) & (iii) we get

$$2\sum(mx_n + c - f_n) = 0$$

$$2\sum(mx_n + c - f_n)x_n = 0$$

By solving The above equations for 'm' & 'c'.

More Simplified Form of above equations.

$$c\sum_1^n 1 + m\sum_1^n x_n = \sum_1^n f_n \quad \text{— (iv)}$$

$$c\sum_1^n x_n + m\sum_1^n x_n^2 = \sum_1^n x_n f_n \quad \text{(v)}$$

Here $\sum_1^n 1$ is simply equal to The number of data points, 'n'

⇒ Exp:- An experiment is carried out and The following data obtained.

x_n	0.24	0.26	0.28	0.30
f_n	1.25	0.80	0.66	0.20

obtained the least square best fit st. line $y = mx + c$ to these data

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give 'm' & 'c' to '2' d.p.

Solution:

Calculate & tabulating the data as

x_n	f_n	x_n^2	$x_n f_n$
0.24	1.25	0.0576	0.3000
0.26	0.80	0.0676	0.2080
0.28	0.66	0.0784	0.1848
0.30	0.20	0.0900	0.0600
$\sum x_n = 1.08$	$\sum f_n = 2.91$	$\sum x_n^2 = 0.2936$	$\sum x_n f_n = 0.7528$

Here. $\sum^{n} 1 = 4$ (i.e For Four points)

Put these values in equation (iv) & (v) we get

$$4c + 1.08m = 2.91 \quad \text{———} (vi)$$

$$1.08c + 0.2936m = 0.7528 \quad \text{———} (vii)$$

By solving the above eq (vi) & (vii)

As ⇒ Multiplying eq (vii) by 3.7037037 on both sides we get

$$\cancel{4c} + 1.087408m = 2.78815$$

Now by eq (vi) $\quad \pm \cancel{4c} \pm 1.08 \quad m = \pm 2.91$

$$-0.007408\ m = 0.12185$$

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$$m = -\frac{0.12185}{0.007408}.$$

$$m = -16.4484$$

$$\Rightarrow m = -16.45$$

put the value of m into eq (v) we get

$$4c + 1.08(-16.45) = 2.91$$

$$4c - 17.76431 = 2.91$$

$$= 2.91 + 17.76431$$

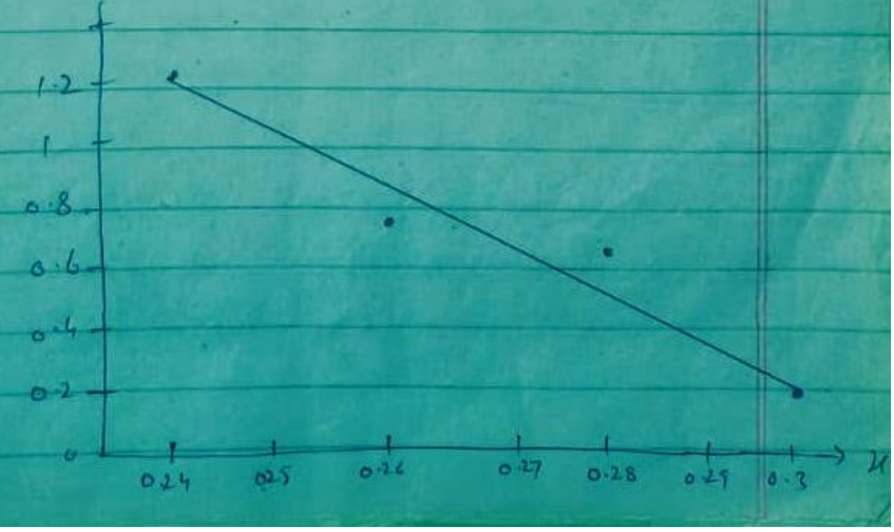
$$4c = 20.67431$$

$$c = 5.1686$$

$$\Rightarrow c = 5.17$$

So the least squares best fit straight line to the given data is

$$y = 5.17 - 16.45x$$



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Assignment Practice Question.

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Exp. #2

Q. Find the best fit straight line to following experimental data

x_n	0.00	1.00	2.00	3.00	4.00
f_n	1.00	3.85	6.50	9.35	12.08

Best of Luck